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Over-injection and self-oscillations in an electron vacuum diode

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We demonstrate a practical means by which one can inject more than the space-charge limiting current into a vacuum diode. This over-injection causes self-oscillations of the space-charge resulting in an electron beam current modulation at a fixed frequency, a reaction of the system to the Coulomb repulsive forces due to charge accumulation. *Published by AIP Publishing.*
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I. INTRODUCTION

Space-charge limited (SCL) emission and flow in charged particle vacuum devices has been the subject of studies for almost a hundred years. A recent review summarizes in detail the advances in this subject since the formulation of the 1D steady state Child-Langmuir (CL) law.[1] Since most practical situations are very different from the 1D infinite parallel plate steady state geometry and the zero initial particle velocity, the prediction of the SCL current amplitude using the CL law is in most cases only indicative. The literature attempting to analytically formulate the SCL of more complex situations than that dealt with the CL law in the steady state is vast.[1]

Birdsall and Bridges[2,3] discovered space-charge instabilities due to virtual cathode (VC) formation in diodes, while being first in developing PIC computational methods. A VC forms when an anode–cathode (AK) gap is over-injected. When too many electrons are injected from the cathode, some of the electron current is returned toward the cathode, and further injection is Coulomb blocked. The location in front of the cathode from where electrons return is almost at cathode potential and is called the VC. The VC emits more current than it acquires because of the smaller distance between the VC and the anode, and the electrons have already gained an initial velocity. When sufficient charge is depleted from the VC, additional charge can be injected into the VC anode gap until again too much charge accumulates and emission is blocked. This process forms a periodic modulation in the space-charge so long as the injection continues. This space-charge modulation can, under certain circumstances, become strong, resulting in periodic electron bunch formation. If over-injection is small, a VC forms, but periodic behavior does not develop. If over-injection is too large, the bunches disappear because of internal Coulomb repulsion. These periodically formed electron bunches survive along the AK gap if the acceleration is strong enough for the bunches to propagate before they disperse due to Coulomb repulsion.

This behavior is qualitatively similar to what is known as VC oscillations and has been the basis for the well-known high-power microwave source, the vircator. Luginsland *et al.*,[4,5] though, have shown that vircator oscillations are possible for finite rise-time pulses producing currents beneath the SCL. This happens because of $-LdI/dt$, where L is the inductance of the experimental system consisting of the coaxial drift tube and the electron beam, which leads to the decrease in the electron energy and, respectively, an increase in the space-charge of the electron beam so that over-injection is formed due to the system inductance.

Attempts to attain self-oscillatory behavior, such as that described earlier without the system induction having a major effect, have been going on for many years. Valfells *et al.*[6] have demonstrated a behavior which they attribute to the formation of a VC when a short pulse of seemingly over-injected charge crosses the AK gap of a diode. The over-injection in this case is thought to be the result of adding photo-emitted electrons by applying a laser pulse to the thermionically emitting surface of a dispenser cathode. Based on these experiments, it was suggested that in microdiodes, it is possible to over-inject the diode by a laser pulse which will be long relative to the transit time of electrons within a gap of the order of μm's. For this case, simulations show that the current in the microdiode self-oscillates in the THz frequency range.[7]

Birdsall and Bridges[2] thought that to over-inject a gap first one needs to accelerate a beam and then before introducing it decelerate it to zero velocity. References 2 and 3, though do not give a practical solution as to how this should be implemented.

Kalinin *et al.*[8] designed a device and performed experiments observing vircator oscillations at relatively low voltages and currents. They designed a diode operating at its SCL current density value, to which a decelerating section is added downstream from a transparent anode. In this triode, the beam reverses direction as it is decelerated which causes a situation similar to "over-injection" near the decelerating electrode, which in turn causes persistent self-oscillations of the electron beam determined by the electron plasma frequency. Moreover, based on this design, a nano-vircator based on a cold cathode consisting of carbon nanotubes was proposed,[9] which is expected to oscillate in the THz regime.

This paper discusses a variant of the Saratov triode where we demonstrate over-injection of the electron current in an accelerating rather than a decelerating gap resulting in self-oscillations of the electron beam. We simulate (using the MAGIC, PIC code[10]) various situations, which we are in the process of investigating experimentally.

II. VARIATIONS ON THE SARATOV TRIODE

The Saratov triode consists of a diode AK gap operating at the space-charge emission limit determined by the voltage V_{A1} applied on a transparent anode relative to a grounded cathode $V_{C1}=0$. The electron beam is guided through a set of electrostatic lenses from a shaped cathode to become a laminar beam in a drift gap between the anode A1 and a third electrode C2 at a voltage V_{C2}. $\Delta V_1=(V_{A1}-V_{C1})>0$ and $\Delta V_2=(V_{C2}-V_{A1})\leq 0$. $\Delta V_2=0$ corresponds to a vircator situation, and the beam is not decelerated at all, and $\Delta V_1=-\Delta V_2$ corresponds to a reflex triode.[11] For the latter, the electrons are in principle decelerated completely. When $\Delta V_1>|\Delta V_2|$, the SCL current developing in gap C1–A1 enters the gap A1–C2, decelerates and returns towards electrode A1 before reaching C2. The accumulated charge near the return point can become too large, in particular, in the presence of a confining axial magnetic field, similarly to an over-injection situation.

A simpler form of the Saratov triode can be seen in Fig. 1. Anode A1 is fully transparent. The distances in each gap area is 10 mm and the voltage on A1 is chosen to be 10 kV. We also apply a constant axial magnetic field of 200 G sufficient for electron magnetization.

In contrast to the Saratov triode, we assume that we can excite a finite emitting area (see the designated red area in Fig. 1) by means of laser photoionization or thermionic emission, so that a uniformly distributed electron current is injected into the system with an amplitude not necessarily at the SCL value. For simplicity, we assume in our PIC simulations that the injection is sudden, the injected electrons start at zero velocity and that the injected current is constant in time. For a diode, such as that of gap C1–A1, we can calculate the steady state SCL current independently by a ray tracing technique.[12] For 10 kV, 10 mm gap, 2.5 mm emission area radius and $B_z=200$ G, we obtain a value of 0.934 A. We first assume that $V_{C2}=V_{C1}=0$, that is, a reflex triode situation. For this case, the current collected on C2 oscillates as seen in Fig. 2. Since the applied voltage and the injected current are constant, the oscillations seen in Fig. 2 are self-oscillations.

As the injected current increases, the amplitudes of the peaks in the collected current increase (Fig. 2). The frequency of the current oscillations in Fig. 2 is ∼2.1 GHz and corresponds to about twice the transit time of a single 10 mm gap. The dynamics responsible for these oscillations is clearly seen in Fig. 3.

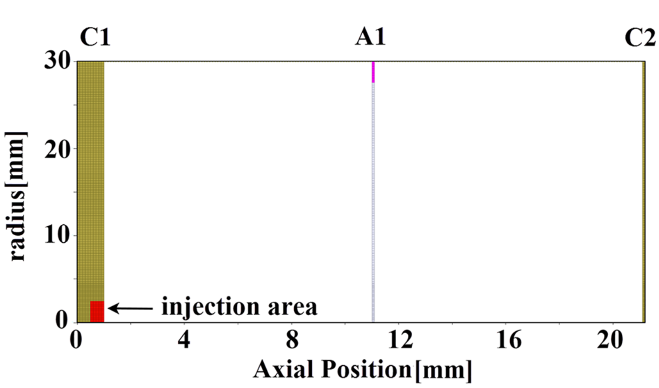

FIG. 1. The geometry of an axially symmetric Saratov triode consisting of 3 electrodes: C_1, A_1, and C_2.

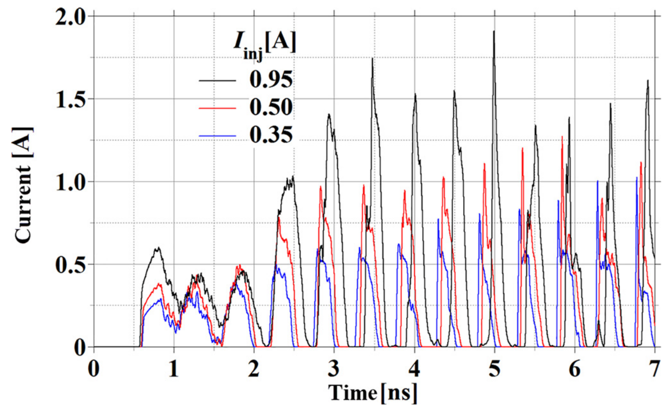

FIG. 2. The current collected on C2 as a function of time for various values of the injected current for $V_{A1}=10$ kV, $V_{C1}=V_{C2}=0$ and $B_z=200$ G.

Figures 3a and 3d are obtained at 5.95 ns which is at a maximum in the collected current in Fig. 2, followed by a trough at 6.2 ns, at which time the snapshots in Figs. 3(c) and 3(d) are taken. This clearly corresponds to the behavior of the beam in both coordinate [Figs. 3(a) and 3(c)] and phase space [Figs. 3(b) and 3(d)]. The beam oscillates between touching C1 and C2. When it touches C2 it deposits current on it which depletes the nearby region from the charge [Figs. 3(a) and 3(b)]. This allows the forward flowing current to fill this depleted region and reduces the return current until the charge becomes too large and repels the forward flowing beam from reaching C2. This behavior produces charge oscillations similar to a VC which is here supplied by a current flowing towards it from the anode rather than being emitted from C2. The return current can approach and touch C1 [Figs. 3(c) and 3(d)] because the injected current is about half the SCL value of the C1–A1 diode, but as soon as the charge density near C1 becomes too large the same process happens as near C2. [Note the continuous injection near C1 in Fig. 3(a)].

Figure 4 shows the distribution of the net axial current along the triode at the same times as the snapshots in Fig. 3. At $t=5.95$ ns (blue curve in Fig. 4), when electrons reach the C2 electrode, space-charge accumulates at that location. The excess charge is already on its way towards C1 (see the negative peak of ∼−1.8 A near $z=8$ mm on the blue curve). At $t=6.2$ ns, the next excess charge is sent backwards from C2 (see the negative peak of ∼−1.6 A near $z=19.5$ mm in the red curve). When excess charge accumulates near C1, this results in a forward electron flow with a current higher than the injected current of 0.5 A. Nevertheless, the average current in each gap does not exceed the SCL current, not even the injected current.

Now, we make $V_{C2}=100$ V, which means that $\Delta V_1>|\Delta V_2|$, that is, in principle, the electrons reach the C2 electrode with a certain velocity instead of zero. The currents collected on C2 for various values of the injected current are seen in Fig. 5.

For $I_{inj}\leq 0.5$ A, the collected current on C2 is constant. Above this value, the current oscillations of the same type as those discussed earlier appear. In Fig. 6, the electron beam for $I_{inj}=0.5$ A is presented. Although the beam spreads in

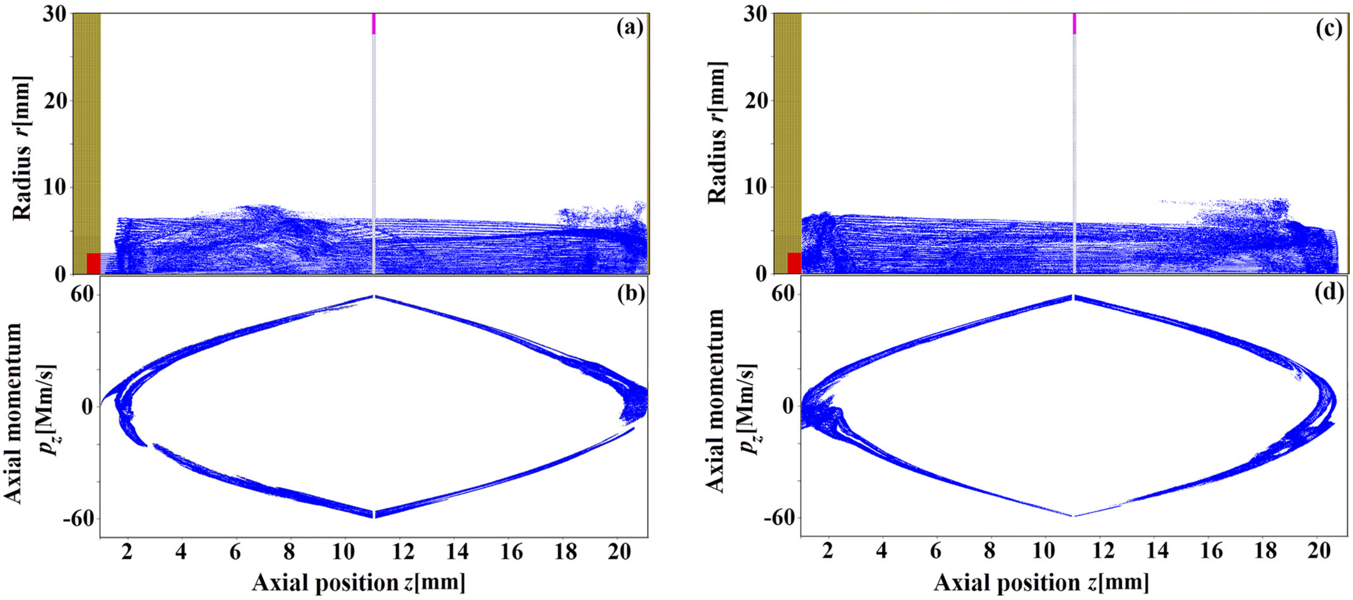

FIG. 3. (a) and (c) Snapshots of electron macroparticle (blue dots) positions in the $[z, r]$ coordinate space and (b) and (d): in the $[z, p_z]$ phase space; at $t = 5.95$ ns in (a) and (b) and 6.2 ns in (c) and (d) for $I_{inj} = 0.5$ A, $V_{A1} = 10$ kV, $V_{C1} = V_{C2} = 0$ and $B_z = 200$ G.

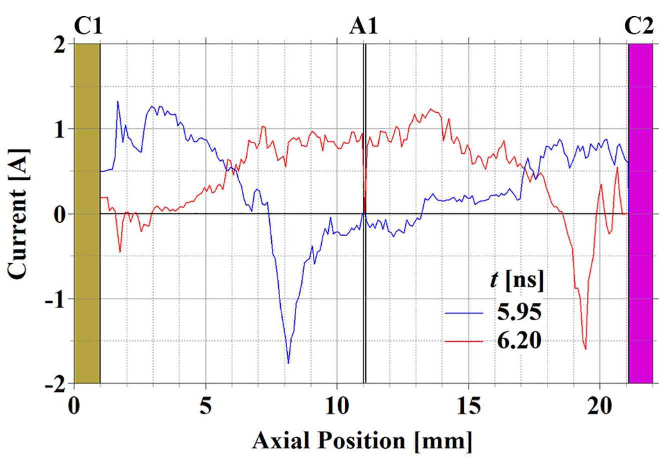

FIG. 4. The axial current distribution along z for $t = 5.95$ ns (blue) and 6.2 ns (red) corresponding to the snapshots in Figs. 3a and 3b, respectively. Positive current means net forward flowing current and negative current is a net backward flowing current.

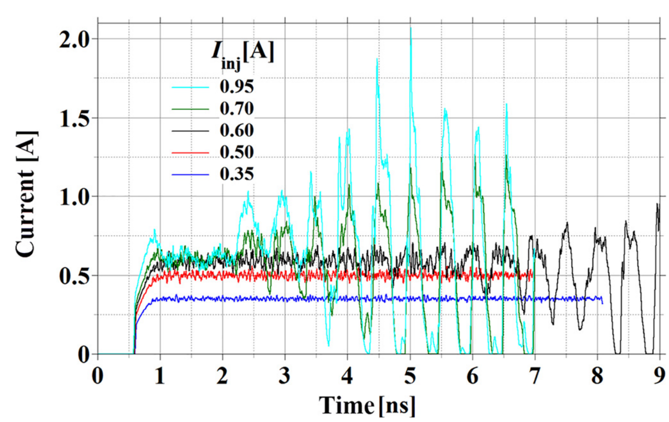

FIG. 5. The current collected on C2 as a function of time for various values of the injected current for $V_{A1} = 10$ kV and $V_{C1} = 0$, $V_{C2} = 100$ V, and $B_z = 200$ G.

radius slightly as it reaches C2, $V_{C2} = 100$ V allows sufficient current to be collected on C2, no over-charge accumulates near C2, no current returns towards A1 and no VC forms in the vicinity of the C2. When the beam in the gap A2–C2 is not completely slowed down, the triode current oscillates only if I_{inj} is above a critical value.

The triode discussed so far, apart from that the C1–A1 gap, has no optical components to ensure that the flow into the A1–C2 gap is laminar, and that the currents considered are below the SCL values corresponding to the applied voltage, is similar to the Saratov triode.[8] The regime of our interest is different than that of Ref. 8 because we are interested in the current collected on C2 which we think can be an over-injection electron source to an additional diode attached downstream to the Saratov triode thus forming a tetrode. This then becomes a method to slow down, not completely, an accelerated beam produced in the C1–A1 gap and

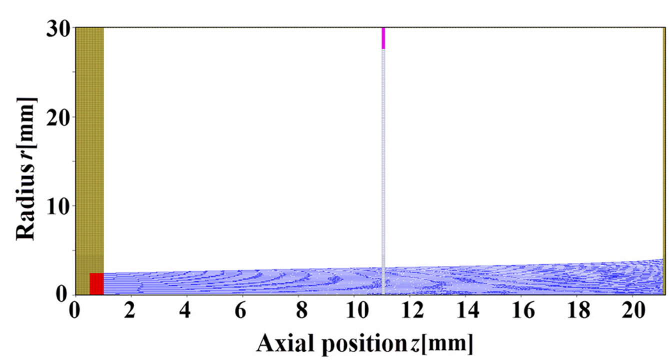

FIG. 6. Snapshot of electron positions at $t = 7$ ns for $V_{A1} = 10$ kV and $V_{C1} = 0$, $V_{C2} = 100$ V, $B_z = 200$ G, and $I_{inj} = 0.5$ A.

introduce it into an additional diode as suggested by Birdsall and Bridges.[2,3] A simpler scheme where an accelerating diode is used directly as the over-injecting source is not possible. For instance, to produce a beam having a radius of 2.5 mm and a current of 0.5 A for an applied voltage of

100 V, one requires an AK gap of 30 μm. The electric fields then reach values >30 kV/cm because of possible non-uniformities that always exist on the cathode surface. The latter can lead to explosive cathode plasma formation and shortening of the AK gap within several nanoseconds due to plasma expansion with typical velocities of 2×10^6 cm/s. Therefore, we suggest the tetrode system where the triode is used as an electron source for the beam injected into the third gap with currents larger than the SCL value for this gap.

III. OVER-INJECTING A DIODE

We attach the additional diode gap to the triode of Fig. 1 by making C2 fully transparent and adding A2 10 mm downstream from it, thus forming a tetrode (Fig. 7). The idea behind this tetrode is to inject into it a nearly slowed down, laminar and continuous electron beam produced in the triode. For the triode C1–A1–C2, when the current produced on C1 is below the SCL of the C1–A1 and C2–A2 gaps, the beam is not oscillatory (see Fig. 6), in contrast to the original Saratov triode which is oscillatory for all conditions considered.[9] We expect that when the current amplitude of this laminar beam is larger than the SCL current of the C2–A2 gap for a given potential difference, we shall obtain self-oscillations as predicted by Birdsall and Bridges.[2,3]

The finite radius, axially symmetric beam entering the diode gap C2–A2 has an initial kinetic energy $E_0 e = mv_0^2/2$ $\sim$100 eV, which corresponds to an initial electron velocity $v_0 = 6 \times 10^9\ cm/s$. For such initial conditions, namely, two parallel plates in an axially symmetric geometry, a finite radius beam and an axial magnetic field, there are no analytical estimates of the SCL current. However, this limit can be estimated for our parameters by calculating the SCL current for zero initial velocities and an axial magnetic field of 200 G, using the ray tracing technique for a beam radius of 4.684 mm taken from the results seen in Fig. 6, and multiplying these by Jaffé's factor[1,13]

$$\Gamma = \left[\left(1 + \frac{E_0}{V_g}\right)^{1/2} + \left(\frac{E_0}{V_g}\right)^{1/2}\right]^3, \qquad (1)$$

where V_g is the C2–A2 gap voltage difference. We calculate by ray tracing the SCL current to be 0.3999 A and 0.2121 A, and $\Gamma = 1.74$ and 1.98, giving $I_{SCL} = 0.696$ A and 0.420, for $V_g = 2.9$ and 1.9 kV ($V_{A2} = 3$ and 2 kV), respectively. Thus, an injected current into the C2–A2 gap of 0.5 A of electrons with an initial energy of 100 eV is smaller than the SCL current for $V_{A2} = 3$ kV, and over-injecting the C2–A2 diode for $V_{A2} = 2$ kV.

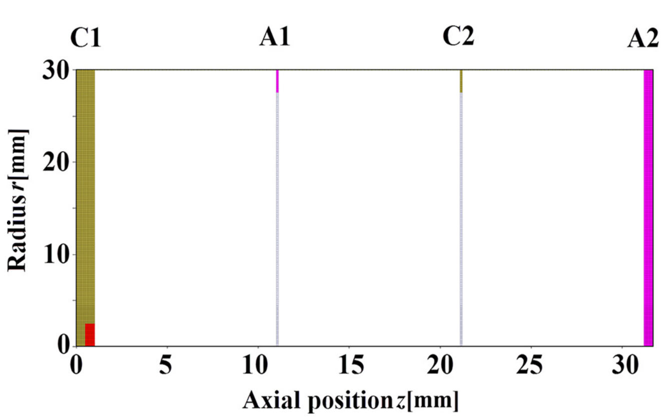

FIG. 7. The geometry of a tetrode consisting of 4 electrodes: C1, A1, C2, and A2.

Let us consider the slowed down, laminar and continuous electron beam of 0.5 A current produced in the triode (Fig. 6) flowing into the C2–A2 gap (Fig. 7). We use the same settings as those used to obtain Fig. 6, that is: $V_{C1} = 0$, $V_{A1} = 10$ kV, $V_{C2} = 100$ V and a fixed axial magnetic field $B_z = 200$ G. In Fig. 7, we apply $V_{A2} = 3$ kV and obtain an electron beam which is first accelerated in gap C1–A1, then decelerated in gap A1–C2 followed by acceleration in gap C2–A2. There is no return current as seen in Fig. 8(b), and the space-charge does not considerably affect the beam radius in the third gap. For this value of V_{A2}, the injected 0.5 A is below the SCL current (0.696 A) and the electron flows unperturbed from the triode into the C2–A2 diode.

The time dependence of the current collected on A2 is shown in Fig. 9. One can see that the amplitude of this is almost the same as the injected 0.5 A, and no current oscillations appear anywhere.

Next, we decrease V_{A2} to 2 kV, and we see in Fig. 10 that strong self-oscillations in the current in the gap C2–A2 develop. Figure 10(c) at $V_{A2} = 2$ kV is to be compared to Fig. 9 at 3 kV. The frequency of the current oscillations in Fig. 10 is the same as the frequency in Fig. 5.

To understand the dynamics, we draw in Fig. 11(a), a snapshot of the electron positions, the electron momentum phase space and the electron net currents in this three-gap tetrode. At $t = 9.4$ ns, when the snapshots in Fig. 11 were taken, the excess charge which has accumulated near C2 is sent forward towards A2 and backwards towards A1 in the triode. This leaves the region near C2 empty of electrons.

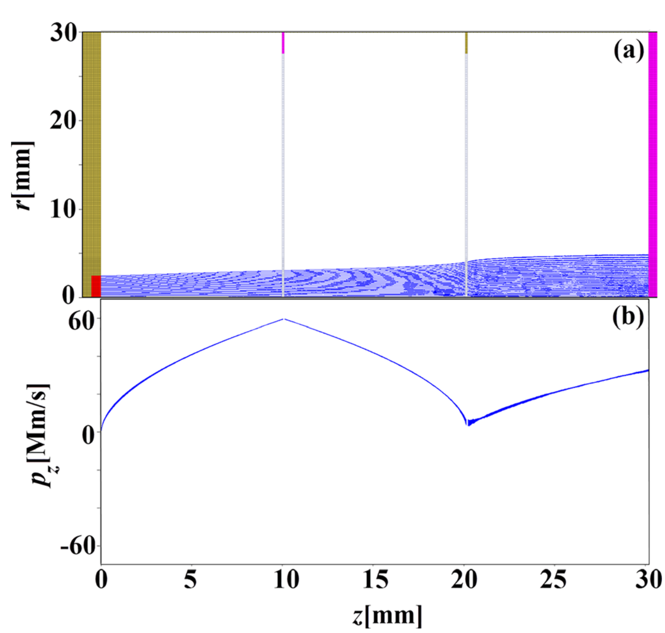

FIG. 8. Snapshot at $t = 9$ ns of electron macroparticle (blue dots) positions in the [z, r] coordinate space (a) and in the [z, p_z] phase space (b) for $I_{inj} = 0.5$ A and $V_{C1} = 0$, $V_{A1} = 10$ kV, $V_{C2} = 100$ V, $V_{A2} = 3$ kV, and $B_z = 200$ G.

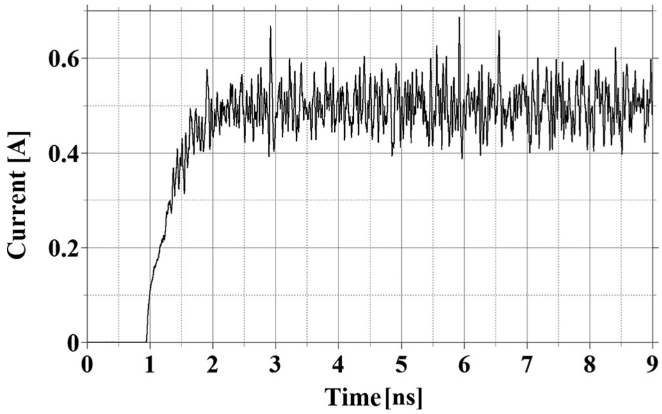

FIG. 9. The current collected on A2 for $V_{A2} = 3$ kV and $I_{inj} = 0.5$ A, $V_{C1} = 0$, $V_{A1} = 10$ kV, $V_{C2} = 100$ V, $V_{A2} = 3$ kV, and $B_z = 200$ G.

[Note: there is no current near C2 in Fig. 11(c).] We also see a bunch propagating in the C2–A2 gap with a peak amplitude exceeding the SCL value (~0.42 A) more than 5 times. Nevertheless, the average current in this gap for any time does not exceed 0.42 A, which suggests that the SCL upper bound is, on average, not violated.

The dynamics of the electron motion in the triode part of the tetrode [Fig. 11(b)] has developed to the same oscillatory dynamics discussed in Sec. II and seen in Fig. 3. The onset of the self-oscillations is not in the triode, but rather it is the result of over-injecting the third gap of the tetrode at the low applied voltage. This becomes clear in the phase space plots seen in Fig. 12.

As long as the charge near C2 is below the SCL, the electrons flow only forward [see Fig. 2(a), $p_z > 0$]. Then, a VC forms in the C2–A2 gap from which part of the current is accelerated towards A2 and some returns towards C2 [see Fig. 2(b), $p_z < 0$]. However, since C2 is transparent, the excess current flows into the A2–C2 gap adding to the charge existing there which makes the initially laminar flow in the C1–A1–C2 triode unstable and oscillatory, followed by oscillatory behavior in the C2–A2 gap too.

The frequency of the current oscillations is governed by the transit time of electrons, ~0.5 ns, from C2 to C1. The

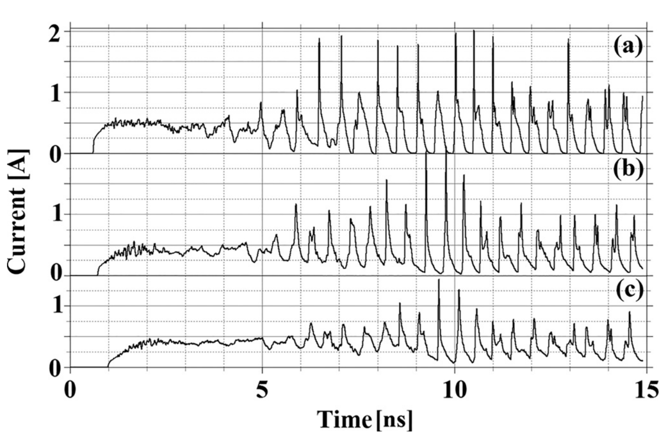

FIG. 10. (a) The current measured at the entry point inside gap C2–A2, (b) at the center of gap C2–A2 and (c) the current collected on A2 for $V_{A2} = 2$ kV and $I_{inj} = 0.5$ A, $V_{C1} = 0$, $V_{A1} = 10$ kV, $V_{C2} = 100$ V, and $B_z = 200$ G.

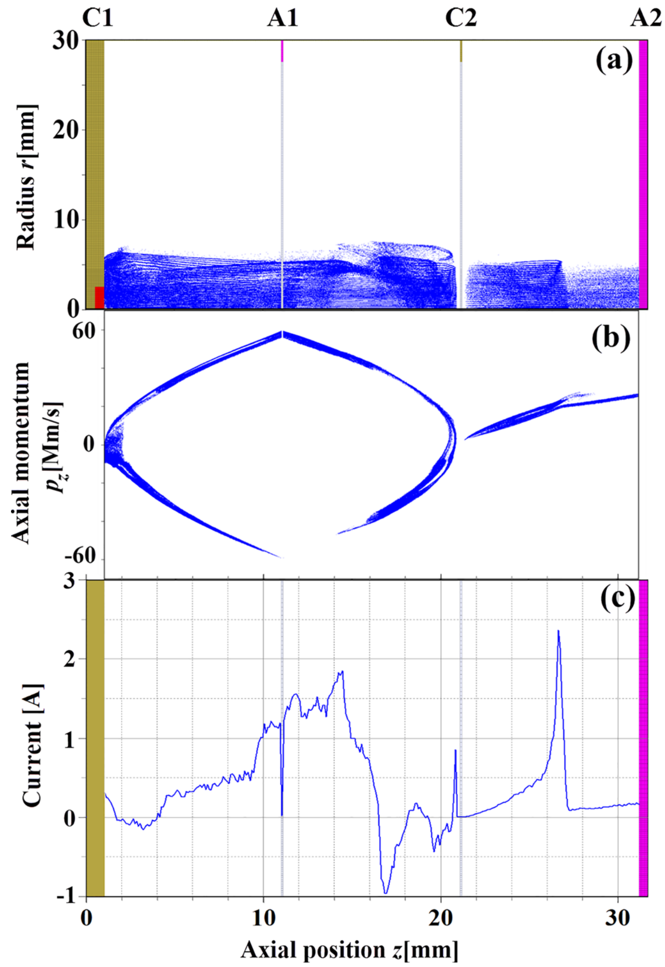

FIG. 11. Snapshot at $t = 9.4$ ns of electron macroparticle (blue dots) positions in the [z, r] coordinate space (a), in the [z, p_z] phase space (b) and the current distribution along z (positive current means net forward flowing current and negative current is a net backward flowing current) (c) for $V_{A2} = 2$ kV and $I_{inj} = 0.5$ A, $V_{C1} = 0$, $V_{A1} = 10$ kV, $V_{C2} = 100$ V, and $B_z = 200$ G.

transit time in C2–A2 for $V_{C2} = 2$ kV is ~0.39 ns. The frequency associated to this transit time does not seem to have a strong effect and does not appear in the FFT of the time traces of Fig. 10. To fully understand the dynamics of the system, a more detailed study of the effect of the distances, the voltages and the injected current needs to be performed.

The tetrode scheme described in this section enables one to over-inject the C2–A2 diode controlled by the potential on A2 and the C2–A2 gap distance. The C2–A2 diode, though, is not an independent entity, over-injecting it affects the dynamics of the triode supplying its current leading to self-oscillations which are governed by the triode parameters.

IV. SUMMARY

Recently, there has been interest in the question of the definition of a steady state SCL current, and whether such a limit consists an upper bound to the SCL current transportable in an unstable diode.[14–16] The system discussed in this paper is a very complex unstable dynamical system even though the voltage is kept constant, and no existing analytical steady state bounds would be applicable for comparison.

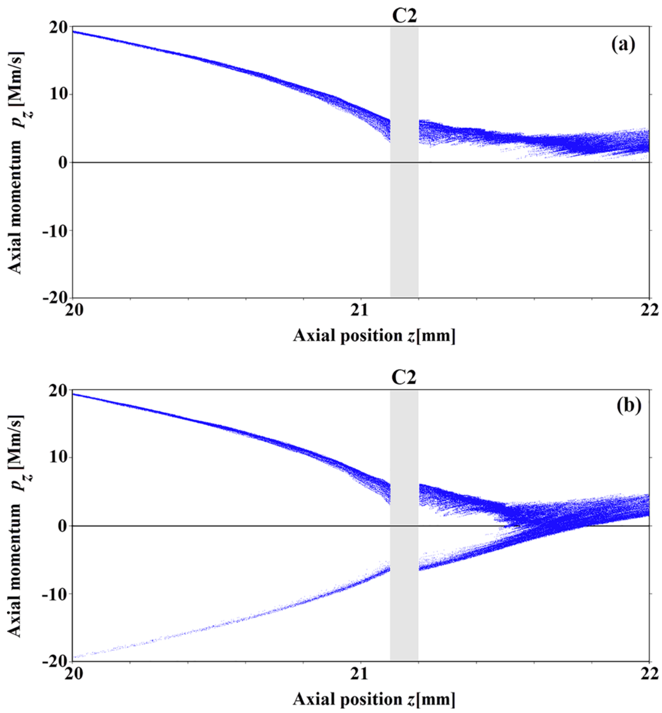

FIG. 12. [z, p_z] phase space plots near the transparent grid C2 (grey area) at (a) $t = 2.1$ ns and (b) 2.5 ns.

On the other hand, Coulomb forces tend to react as soon as too much charge accumulates in various regions at certain time intervals which means that the dynamics is the result of a SCL upper bound.

We demonstrated that it is possible to over-inject a diode by an externally decelerated electron beam and that indeed this causes the system to become unstable, and therefore current oscillations develop. The amplitudes of these current bunches are higher than the SCL current of the diode, but this does not mean that the SCL upper bound has been violated.

The parameters chosen in the numerical experiments presented above are not unique. For an experiment, one has also got to consider that the injection in practice is not sudden and that the pulse needs to be long enough for the oscillations to develop. A1 and A2 electrodes are in practice not completely transparent and scattering will occur too. Experimentally, one may need to make A2 transparent and reverse the signs of the applied potentials in order to collect the electron bunches formed in the C2–A2 gap. We are in the process of building an experimental setup where we shall be able to observe the behavior predicted in this paper.

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