Self-Injection-Locked Magnetron as an Active Ring Resonator Side Coupled to a Waveguide With a Delayed Feedback Loop

Y. P. Bliokh, Y. E. Krasik, and J. Felsteiner

Abstract—**Theoretical analysis and numerical simulations of the magnetron operation with a feedback loop were performed assuming that the delay of the electromagnetic wave propagating in the loop is constant whereas the phase of the complex feedback reflection coefficient is varied. Results of simulations showed that, by a proper adjustment of the values of the time delay and phase of the reflection coefficient that determines phase matching between the waves in the resonator and feedback loop, one can increase the magnetron output power significantly without any other additional measures.**

Index Terms—**Feedback circuits, magnetrons.**

I. Introduction

MAGNETRON oscillators are widely used sources of microwave radiation operating over a wide range of powers and frequencies and having various applications [1]. A typical feature of the magnetron operation is a relatively broad and noisy spectrum [2]. However, for many applications (radar, communication systems, charged-particle accelerators, etc.), it is very necessary both to narrow the frequency spectrum and to reduce the noise. Improvements in the microwave spectrum can be achieved either by using a special magnetron design, which makes starting conditions more preferable for the operating mode, or by feeding external monochromatic electromagnetic radiation into the magnetron cavity. In the latter case, this controlling radiation stabilizes the magnetron operation similarly to other nonlinear oscillators [3]. In fact, the injected electromagnetic radiation can be produced either by an external source (injection-locked magnetron (ILM) [4]–[7]) or by the same magnetron (self-inject-locked magnetron (SILM) [8]). In the latter case, a part of the magnetron output power is injected back into the magnetron cavity via a feedback loop. It is worth noting that this feedback loop can appear as a result of the magnetron radiation reflection from the load. It has been shown [4]–[8] that ILMs and SILMs are able to fix the operating frequency and phase and essentially decrease the noise of the radiation.

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The authors are with the Department of Physics, Technion, 32000 Haifa, Israel (e-mail: bliokh@physics.technion.ac.il; fnkrasik@physics.technion.ac.il; josh@physics.technion.ac.il).

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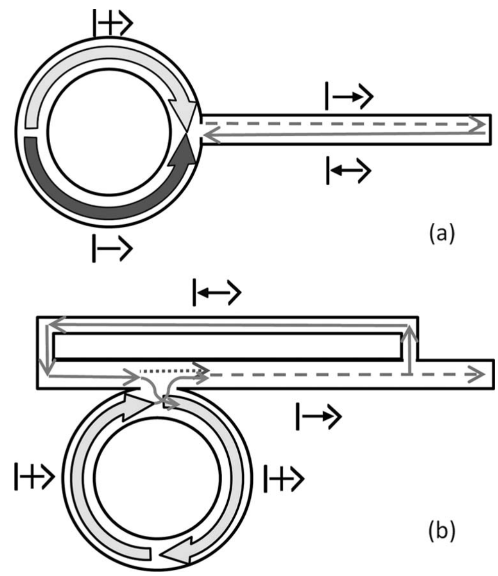

Fig. 1. Magnetron (ring resonator) and waveguide. (a) Clockwise $|+\rangle$ and (light-gray bold arrow) and anti-clockwise $|-\rangle$ (dark-gray bold arrow) propagating eigenmodes. Direct (dashed line) and reflected (solid line) waves in the waveguide. (b) Ring resonator side-coupled to a waveguide with a feedback loop.

Recent investigations [9], [10] show that the efficiency of the relativistic S-band π-mode-operated magnetron can be significantly (up to 40%) increased when part of the radiated electromagnetic power is reflected into the magnetron cavity. This effect, to our knowledge, has not yet been discussed. In this paper, we suggest an explanation for this effect using a model of the magnetron which considers it as an active nonlinear ring resonator. This model can be applied also to coupled ring resonator–fiber systems which are of interest for optoelectronics and communication (see, e.g., [11] and [12]).

II. Geometry of the Problem

In the model, a magnetron is considered as a ring resonator (see Fig. 1). The electromagnetic field of the magnetron operating mode (π-mode) can be considered as a standing wave presented as a sum of two, clockwise $|+\rangle$ and anticlockwise $|-\rangle$, propagating waves having equal frequencies ω and equal amplitudes $A_{|+\rangle} = A_{|-\rangle}$. In spite of the fact that only one wave (wave $|+\rangle$, for definiteness) is excited due to the resonant interaction with the rotating electron flow when the electron azimuthal drift velocity is close to the wave phase velocity, the

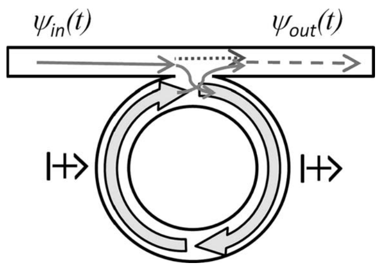

Fig. 2. Auxiliary problem. Phase matched coupling between waveguide and waveguide modes. Notation is the same as in Fig. 1.

π-mode keeps its structure, i.e., the parity of amplitudes $A_{|+\rangle}$ and $A_{|-\rangle}$. Thus, one can consider one wave ($|+\rangle$, for instance) only, keeping in mind that the second, $|-\rangle$ wave, has the same amplitude. The energy stored in the resonator leaks partially into the waveguide [see Fig. 1(a)].

The output end of the waveguide is connected to a certain payload, for instance, an antenna. It is assumed that the matching between the waveguide and the load is not ideal, i.e., part of the wave energy is reflected and enters back into the magnetron cavity. This reflected wave excites two contrapropagating waves, $|+\rangle$ and $|-\rangle$, in the magnetron cavity. Due to the aforementioned relation between the amplitudes of these waves, one can consider the active $|+\rangle$ wave only which is amplified by the electron flow. By convention, the paths of rightward and leftward propagating waveguide modes ($|\rightarrow\rangle$ and $|\leftarrow\rangle$ modes) can be considered as separated paths. Namely, the wave which is reflected from the waveguide output end comes back to the resonator along a *feedback loop* [see Fig. 1(b)]. The separation of the $|\rightarrow\rangle$ and $|\leftarrow\rangle$ mode paths means that the reflection of the $|\leftarrow\rangle$ mode from the waveguide–resonator coupling element is absent by definition. Thus, we finally arrive at the model of the ring resonator *side coupled* to the waveguide, and the coupling between the resonator and waveguide modes is a *phase matched* coupling. This means that only the copropagating waves are coupled: Modes $|\rightarrow\rangle$ and $|+\rangle$ (modes $|\leftarrow\rangle$ and $|-\rangle$) are coupled, whereas modes $|\rightarrow\rangle$ and $|-\rangle$ (modes $|\leftarrow\rangle$ and $|+\rangle$) are not coupled. The phase matched coupling guarantees the absence of the $|\leftarrow\rangle$ mode reflection mentioned earlier. This configuration, i.e., side-coupled waveguide and ring (or disk) resonator, has widely been explored in photonics (see, e.g., [11] and [12]). The general properties of this system were reported in [13], and the results of this work will be used in the next section.

III. Small Signal Model

Let us consider a supplementary problem, namely, wave propagation in the waveguide coupled with a ring resonator (see Fig. 2). Let $\psi_{in,\omega}$ be the Fourier transform of the rightward propagating waveguide mode $\psi_{in}(t)$ on the left side with respect to the waveguide–resonator coupling cross section. The Fourier transform $\psi_{out,\omega}$ of the transmitted wave $\psi_{out}(t)$ on the right side with respect to this cross section and $\psi_{in,\omega}$ are related as

$$\psi_{out,\omega} = t(\omega)\psi_{in,\omega} \tag{1}$$

where the transmission coefficient $t(\omega)$ is defined as [13]

$$t(\omega) = 1 - i\frac{1}{\omega - \omega_r + i\Gamma}\frac{L}{v_g}|V_{r,w}|^2. \tag{2}$$

Here, ω_r is the eigenfrequency of the ring resonator, $\Gamma = \Gamma_d + \Gamma_c$, the coefficient Γ_d is related to the resonator dissipative losses, the coefficient $\Gamma_c = |V_{r,w}|^2 L/2v_g$ is related to the energy leakage from the resonator because of coupling with a waveguide characterized by the coupling coefficient $V_{r,w}$, L is the resonator perimeter, and v_g is the group velocity of the resonator eigenmode. Equation (2) is valid when the coupling is weak

$$\Gamma_c/\omega_r \ll 1. \tag{3}$$

Parameters Γ, Γ_d, and Γ_c can be written in terms of the resonator Q-factors: $\Gamma = \omega_r/2Q_{tot}$, $\Gamma_d = \omega_r/2Q_{diss}$, and $\Gamma_c = \omega_r/2Q_{leak}$. Here, Q_{tot} is the total Q-factor, and Q_{diss} and Q_{leak} are the Q-factors responsible for the energy dissipation and leakage, respectively. A *negative* dissipative Q-factor, $Q_{diss} < 0$, corresponds to an *active* resonator, which amplifies an incoming wave.

The output wave is a superposition of two waves, namely, the wave which crosses the coupling region directly through the waveguide and the wave that enters the waveguide from the resonator. The two terms in the right side of (2) describe these two paths of the wave transmission. The first term, unity, describes the wave's direct transmission through the waveguide[1]; the second term describes the wave's transmission through the resonator. Thus, $\psi_{out,\omega}$ is the sum of two terms

$$\psi_{out,\omega} = t(\omega)\psi_{in,\omega} = \psi_{direct,\omega} + \psi_{res,\omega} \tag{4}$$

where $\psi_{direct,\omega} \simeq \psi_{in,\omega}$ is the Fourier transform of the wave transmitted directly through the waveguide and $\psi_{res,\omega}$ is the Fourier transform of the wave transmitted through the resonator:

$$\begin{aligned}\psi_{res,\omega} &= [t(\omega) - 1]\,\psi_{in,\omega}\\ &= -\frac{iQ_{leak}^{-1}}{\omega/\omega_r - 1 + i\left(Q_{diss}^{-1} + Q_{leak}^{-1}\right)/2}\psi_{in,\omega}.\end{aligned} \tag{5}$$

Presenting (5) in the form

$$\left[-i\omega + i\omega_r + \omega_r\left(Q_{diss}^{-1} + Q_{leak}^{-1}\right)/2\right]\psi_{res,\omega} = -\omega_r Q_{leak}^{-1}\psi_{in,\omega}$$

one can see that this is the Fourier transformation of the differential equation

$$\begin{aligned}\frac{d\psi_{res}(t)}{dt} + i\omega_r\psi_{res}(t) + \frac{\omega_r}{2}\left(\frac{1}{Q_{leak}} + \frac{1}{Q_{diss}}\right)\psi_{res}(t)\\ = -\frac{\omega_r}{Q_{leak}}\psi_{in}(t).\end{aligned} \tag{6}$$

[1] The fraction $\sim Q_{leak}^{-1}$ of the incoming wave energy passes through the resonator, and therefore, the corresponding transmission coefficient t_w is not equal to unity. This fraction is small by virtue of condition (3) and can be neglected because both terms in (2) are of the same order of magnitude, of the order of unity.

Thus, one can write the following equation for input and output waves having arbitrary shapes:

$$\psi_{out}(t) = \psi_{in}(t) + \psi_{res}(t) \tag{7}$$

where $\psi_{res}(t)$ is the solution of (6). Here, let us remind that the wave $\psi_{res}(t)$ is the wave that leaks from the resonator into the waveguide. The wave field in the resonator can be significantly larger than the wave field $\psi_{res}(t)$ in the waveguide [weak coupling between the resonator and waveguide, see (3)]. However, these wave fields are linearly proportional to each other, and the field $\psi_{res}(t)$ can be considered as a normalized field in the resonator.

A supplementary problem that is being considered helps one to describe the system with the presence of a feedback loop [see Fig. 1(b)]. In a general case, the wave propagation along the feedback loop is described by a functional equation

$$\psi_{in}(t) = \hat{F}\left\{\psi_{out}(t)\right\} \tag{8}$$

where $\hat{F}$ is a certain functional. In the simplest case of a nondispersive feedback loop, this relation has the form

$$\psi_{in}(t) = q\psi_{out}(t-T) \tag{9}$$

where T is the time of the wave propagation along the loop (delay time) and q is the complex feedback coefficient, $|q| < 1$. Hereinafter, we will consider this simple model only.

Equations (6), (7), and (9) [(8) in the general case] being supplemented by initial conditions are a closed system that describes the small signal theory of a magnetron with a delayed feedback. Asymptotic solutions of these equations are rather simple. There is an exponentially either growing or decaying output wave amplitude, depending on whether the system's self-excitation threshold is exceeded or not. In order to describe the wave amplitude saturation, nonlinear terms should be incorporated in these equations.

IV. Nonlinear Model

Without the feedback loop, i.e., when $q = 0(\psi_{in} = 0)$, (6) describes a linear resonator with the self-excitation threshold defined as

$$Q_{diss} + Q_{leak} = 0. \tag{10}$$

When $Q_{diss}^{-1} + Q_{leak}^{-1} < 0$, the field amplitude $\psi_{res}(t)$ grows exponentially. The increase in the amplitude can be limited by the replacement of the *linear* dissipation term $Q_{diss}^{-1} < 0$ in (6) by the *nonlinear* one: $Q_{diss}^{-1} \rightarrow \tilde{Q}_{diss}^{-1} = Q_{diss}^{-1}(1 - |\psi_{res}(t)|^2/\gamma_d^2)$, where γ_d is the parameter that is related to the saturation amplitude. With nonlinear Q_{diss}^{-1}, (6) takes the form

$$\frac{d\psi_{res}(t)}{dt} + i\omega_r\psi_{res}(t) + \frac{\omega_r}{2}\left[\frac{1}{Q_{leak}} + \frac{1}{Q_{diss}}\left(1 - \left|\psi_{res}(t)\right|^2/\gamma_d^2\right)\right]\psi_{res}(t) = 0. \tag{11}$$

This is the Van der Pol equation which is widely used as a model of a nonlinear auto-oscillating dynamical system [14]. Another effect which, in general, should be taken into account is a nonlinear shift of the resonator eigenfrequency. This effect can be incorporated for in the model by the replacement in (6) $\omega_r \rightarrow \omega_r(1 - |\psi_{res}(t)|^2/\gamma_f)$, where γ_f is the parameter that is related to the frequency shift. The latter results in describing the Duffing oscillator. Both Van der Pol and Duffing models are typical examples of nonlinear 2-D oscillating systems [14]. Duffing's model describes nonisochronal oscillations, and Van der Pol's model describes a limit cycle when the equilibrium point loses its stability. It is reasonable to suppose that, due to their generality, these models can be used to describe the properties of the system under consideration. To summarize, let us present hereinafter the equations which will be analyzed

$$\frac{d\psi_{res}(t)}{dt} + i\omega_r\left(1 - \left|\psi_{res}(t)\right|^2/\gamma_f\right)\psi_{res}(t) + \frac{\omega_r}{2}\left[\frac{1}{Q_{leak}} + \frac{1}{Q_{diss}}\left(1 - \left|\psi_{res}(t)\right|^2/\gamma_d^2\right)\right]\psi_{res}(t) = -\frac{\omega_r}{Q_{leak}}\psi_{in}(t), \tag{12}$$

$$\psi_{in}(t) = q\psi_{out}(t-T), \tag{13}$$

$$\psi_{out}(t) = \psi_{in}(t) + \psi_{res}(t). \tag{14}$$

V. Analysis

One can rewrite (12)–(14) in the dimensionless form using dimensionless time $\tau = \omega_r t$ and normalized amplitudes $A_i(\tau) = \psi_i(\tau)e^{i\tau}\gamma_d^{-1}\sqrt{Q_{leak}/(Q_{leak} - |Q_{diss}|)}$

$$\frac{dA_{res}(\tau)}{d\tau} - i\kappa|Q_{diss}|\frac{\gamma_d^2}{\gamma_f}\left|A_{res}(\tau)\right|^2 A_{res}(\tau) - \frac{1}{2}\kappa\left[1 - \left|A_{res}(\tau)\right|^2\right]A_{res}(\tau) = -\frac{1}{Q_{leak}}A_{in}(\tau) \tag{15}$$

$$A_{in}(\tau) = qe^{-i\theta}A_{out}(\tau - \theta), \tag{16}$$

$$A_{out}(\tau) = A_{in}(\tau) + A_{res}(\tau), \tag{17}$$

Here, $\theta = \omega_r T$ is the dimensionless delay time and $\kappa = (Q_{leak} - |Q_{diss}|)/Q_{leak}|Q_{diss}|$. In (15), it was taken into account that $Q_{diss} < 0$.

Equations (15)–(17) have monochromatic solutions with constant amplitudes. Setting $A_i(\tau) = B_i e^{-i\nu\tau}$, where $\nu = (\omega - \omega_r)/\omega_r$ is the normalized frequency shift, we arrive at the following set of algebraic equations:

$$-i\nu B_{res} - i\kappa|Q_{diss}|\frac{\gamma_d^2}{\gamma_f}|B_{res}|^2 B_{res} - \frac{1}{2}\kappa\left(1 - |B_{res}|^2\right)B_{res} = -\frac{1}{Q_{leak}}B_{in}, \tag{18}$$

$$B_{in} = qe^{-i\theta + i\nu\theta}B_{out}, \tag{19}$$

$$B_{out} = B_{in} + B_{res}. \tag{20}$$

Excluding B_{in}, one can obtain

$$i\left(\nu+\kappa|Q_{diss}|\frac{\gamma_d^2}{\gamma_f}|B_{res}|^2\right)+\frac{1}{2}\kappa\left(1-|B_{res}|^2\right) = \frac{1}{Q_{leak}}\frac{qe^{-i\theta+i\nu\theta}}{1-qe^{-i\theta+i\nu\theta}}. \quad (21)$$

Equation (21) determines both frequency shift ν and normalized power in the resonator $P=|B_{res}|^2$ ($P=1$ without the feedback, $q=0$). In the simple case when the nonlinear dispersion is absent ($\gamma_f\to\infty$), one can rewrite the complex equation (21) as

$$\nu = -\frac{|q|}{Q_{leak}}\frac{\sin(\theta-\nu\theta-\alpha)}{1+|q|^2-2|q|\cos(\theta-\nu\theta-\alpha)}, \quad (22)$$

$$P=|B_{res}|^2 = 1-\frac{2|q|}{\kappa Q_{leak}}\frac{\cos(\theta-\nu\theta-\alpha)-|q|}{1+|q|^2-2|q|\cos(\theta-\nu\theta-\alpha)}, \quad (23)$$

where $\alpha=\arg\,q$ is the phase of the feedback coefficient.

As it follows from (23), the normalized power can be varied in the range

$$1-\frac{2|q|}{1-|q|}\frac{1}{\kappa Q_{leak}}\le P\le 1+\frac{2|q|}{1+|q|}\frac{1}{\kappa Q_{leak}}. \quad (24)$$

When the left boundary in (24) is negative, the minimal possible power is equal to zero.

Near the self-excitation threshold, the parameter $\kappa Q_{leak}=(Q_{leak}-|Q_{diss}|)/|Q_{diss}|$ is small. In the latter case, the feedback can noticeably increase the power P in the resonator. The inequalities (24) determine *potentially possible* boundaries of the power variation. Upper and lower boundaries correspond to such values of the frequency shift ν for which $\cos(\theta-\nu\theta-\alpha)=\mp 1$, respectively. However, at designated values θ and α, the frequency ν is determined by the solution of the *dispersion equation* (22). The proximity of the power value to the upper possible value depends on whether the solutions of the dispersion equation contain eigenfrequencies ν for which $\cos(\theta-\nu\theta-\alpha)$ is rather close to -1.

It is convenient to present the dispersion equation (24) as

$$\varphi=\theta-\alpha+\frac{\theta|q|}{Q_{leak}}\frac{\sin\varphi}{(1+|q|^2-2|q|\cos\varphi)} \quad (25)$$

where $\varphi=\theta-\alpha-\theta\nu$. If the feedback coefficient is small, $|q|^2\ll 1$, and

$$\theta<Q_{leak}/|q|, \quad (26)$$

equation (25) has one solution $\varphi(\alpha)$ (assuming delay time $\theta=\mathrm{const}$). The inequality (26) will be considered as a *short* delay time condition. In the case of a reflection coefficient phase α variation, the value $\cos[\varphi(\alpha)]$ varies in a full range from -1 to $+1$. Accordingly, the power P varies in a full range defined by (23). In the opposite case of a *long* delay time, when

$$\theta\gg Q_{leak}/|q|, \quad (27)$$

the dispersion equation (25) has many roots, and the eigenfrequency ν for which $\cos[\theta-\nu(\alpha)\theta-\alpha]$ is close to -1 is always present. This means that (22) and (23) always have solution for which the power P value is close to its upper limit defined by (24).

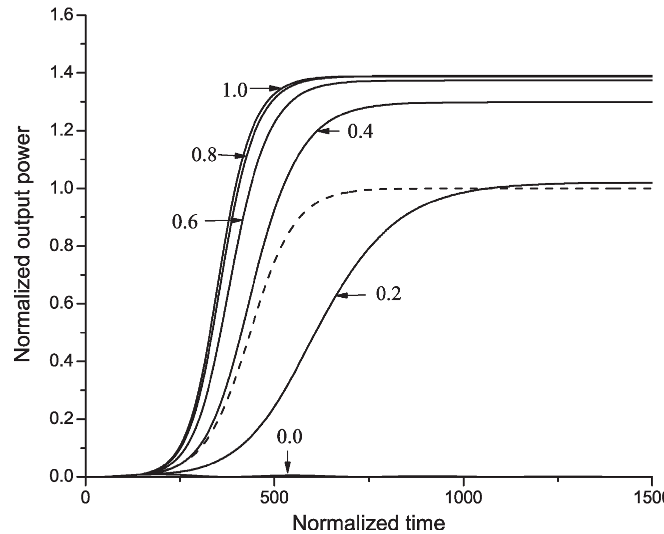

Fig. 3. Dependence of the normalized power on time. "Short" delay time $\theta<Q_{leak}/|q|$. Dashed line corresponds to $q=0$. The curves are marked by corresponding α/π values. $|Q_{diss}|=15$, $Q_{leak}=20$, $q=0.2$, and $\theta=70$.

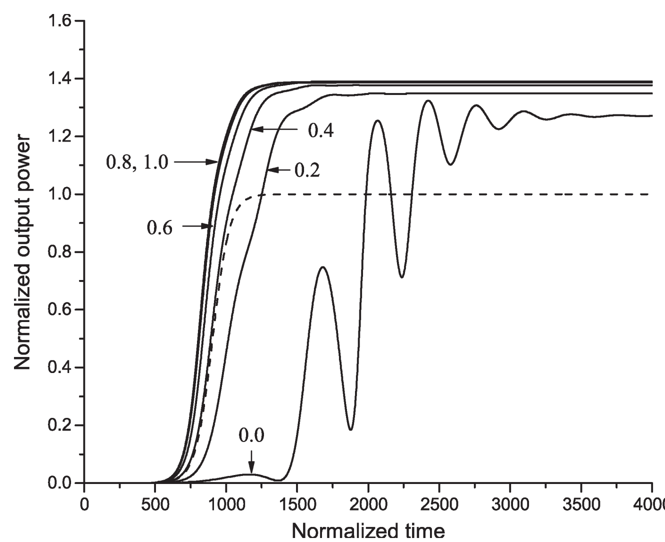

Fig. 4. Same as in Fig. 3. "Long" feedback loop, $\theta\gg Q_{leak}/|q|$. $|Q_{diss}|=15$, $Q_{leak}=20$, $q=0.2$, and $\theta=200$.

The analysis presented earlier shows that a variation in the reflection coefficient phase can produce a rather strong power variation if the inequality (26) is satisfied. In the opposite case, when the inequality (27) is satisfied, the power is close to its maximal possible value, which can noticeably exceed the power in the resonator without a feedback loop.

VI. Numerical Simulations

Examples of numerical solutions of (15)–(17) are presented in Figs. 3 and 4.

Fig. 3 demonstrates how the power saturation level depends on the phase α when the inequality (26) is satisfied. Parameters used for numerical simulation were chosen close to their estimations for the experimental setup described in [9] and [10]:

$|Q_{diss}| = 15$, $Q_{leak} = 20$, $q = 0.2$, and $\theta = 70$. The results of simulations showed that the microwave generation in the magnetron can be either totally suppressed $(\alpha = 0)$ or enhanced by ∼40% $(\alpha \simeq \pi)$ as compared with the magnetron operation without feedback $(q = 0)$.

Fig. 4 is related to the "long" feedback loop when the inequality (27) is satisfied. In agreement with the analysis presented earlier, the power saturation level is almost independent of the phase α and is close to its maximal value for the system with a "short" feedback loop (short delay time).

VII. Discussion

The theoretical analysis and numerical simulations of the magnetron operation with a feedback loop were performed assuming that the time θ of the wave propagating in the feedback loop is constant whereas the phase α of the reflection coefficient q is varied. These assumptions were used only to simplify the solution. In general, the solution of the dispersion equation (25) depends on both phase α and delay time θ. Variation in the value of θ affects the solution in the same manner as the phase α variation. This similarity becomes important particularly when $\theta \gg \pi$ and the condition (26) is satisfied. Thus, the length of the waveguide that connects the magnetron with a payload (an antenna, for instance) can affect the magnetron characteristics essentially. Results of simulations showed that, by a proper adjustment of the values of θ and α that determines phase matching between the waves in the resonator and feedback loop, one can increase the magnetron's output power significantly without any other additional measures.

References

[1] J. Benford, J. A. Swegle, and E. Schamiloglu, *High Power Microwaves*, 2nd ed. New York: Taylor & Francis, 2007.

[2] K. Kanamoto, H. Kuronuma, T. Koinuma, and N. Teshiro, "A study of magnetron noise," *IEEE Trans. Electron Devices*, vol. ED-34, no. 5, pp. 1223–1226, May 1987.

[3] R. Adler, "A study of locking phenomena in oscillators," *Proc. IRE*, vol. 34, no. 6, pp. 351–357, Jun. 1946.

[4] R. D. Weglein and H. A. Leach, "The noise behavior of an injection-locked magnetron reflection amplifier," in *Proc. IEEE MTT-S Int. Microw. Symp. Digest*, Las Vegas, NV, Jun. 9–11, 1987, vol. 1, pp. 261–264.

[5] T. A. Treado, R. A. Bolton, T. A. Hansen, P. D. Brown, and J. D. Barry, "High-power, high efficiency, injection-locked, secondary-emission magnetron," *IEEE Trans. Plasma Sci.*, vol. 20, no. 3, pp. 351–359, Jun. 1992.

[6] I. Tahir, A. Dexter, and R. Carter, "Frequency and phase modulation performance of an injection-locked CW magnetron," *IEEE Trans. Electron Devices*, vol. 33, no. 7, pp. 1721–1729, Jul. 2006.

[7] A. C. Dexter, G. Burt, R. G. Carter, I. Tahir, H. Wang, K. Davis, and R. Rimmer, "First demonstration and performance of an injection locked continuous wave magnetron to phase control a superconducting cavity," *Phys. Rev. Special Topics, Accel. Beams*, vol. 14, no. 3, pp. 032001-1–032001-17, Mar. 2011.

[8] J. J. Choi and G. W. Choi, "Experimental observation of frequency locking and noise reduction in a self-injection-locked magnetron," *IEEE Trans. Electron Devices*, vol. 54, no. 12, pp. 3430–3432, Dec. 2007.

[9] A. Sayapin, Y. Hadas, and Y. E. Krasik, "Drastic improvement in the S-band relativistic magnetron operation," *Appl. Phys. Lett.*, vol. 95, no. 7, pp. 074101-1–074101-3, Aug. 2009.

[10] A. Sayapin and A. Shlapakovski, "Transient operation of the relativistic S-band magnetron with radial output," *J. Appl. Phys.*, vol. 109, no. 6, pp. 063301-1–063301-5, Mar. 2011.

[11] B. E. Little, S. T. Chu, H. A. Haus, J. Foresi, and J.-P. Laine, "Microring resonator channel dropping filters," *J. Lightw. Technol.*, vol. 15, no. 6, pp. 998–1005, Jun. 1997.

[12] J. M. Choi, R. K. Lee, and A. Yariv, "Control of critical coupling in a ring resonator–fiber configuration: Application to wavelength-selective switching, modulation, amplification, and oscillation," *Opt. Lett.*, vol. 26, no. 16, pp. 1236–1238, Aug. 2001.

[13] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, "Scattering-theory analysis of waveguide–resonator coupling," *Phys. Rev. E*, vol. 62, no. 5, pp. 7389–7404, Nov. 2000.

[14] J. Guckenheimer and P. Holmes, *Nonlinear Oscillators, Dynamical Systems, and Bifurcations of Vector Fields*. New York: Springer-Verlag, 1990.

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